

Automatically biasing tangent vectors for use with cubic hermite interpolation.

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1 Introduction

Cubic interpolation allows us to produce a smoothly interpolated line based on a set of points. The result of this function can be used in many areas, from camera tracking to terrain smoothing. We use the cubic hermite[1] function to interpolate between two points with given tangent vectors, which provides the basis for the construction of splines[2] using cubic interpolation.

$$p_n(t) = (2t^3 - 3t^2 + 1)p_n + (t^3 - 2t^2 + t)m_n + (-2t^3 + 3t^2)p_{n+1} + (t^3 - t^2)m_{n+1}$$

Appropriate tangent vectors can be calculated using a number of methods, for example the Catmull-Rom[3] method:

$$m_i = \frac{1}{2}(p_{i+1} - p_{i-1})$$

This gives nice results in many situations, but there are often situations where the result is not so good, and more complex tangent functions can be used.

2 Biasing Tangents Algorithm

We can use a biased tangent function to calculate a different set of values for mu.

$$m_i = \text{normalize}(p_j - p_k) \times \text{bias} \\ (j \neq k)$$

You can produce an equivalent bias that will produce the output of the Catmull-Rom method:

$$m_i = \text{normalize}(p_{i+1} - p_{i-1}) \times \frac{1}{2} \|p_{i+1} - p_{i-1}\|$$

Obviously, the computational cost will be much more expensive in this simple case, and you should not use this method for producing this output. However, it can be seen that the bias value is separate from the tangent calculation, which allows for much more flexibility in the final result, as you can manipulate the bias value separately.

This technique can be applied to many problems. For example, when building a spline to connect two tubes, only the length of the mid-section should have an effect on the curvature, so we define the following function for drawing a spline between the center two points of a four point set:

$$\begin{aligned} m_1 &= \text{normalize}(p_1 - p_0) \times \|p_2 - p_1\| \\ m_2 &= \text{normalize}(p_2 - p_3) \times \|p_2 - p_1\| \end{aligned}$$

Using these functions, it is possible to get a consistently round output from an interpolated connective joint, as the tangents will scale based on the length between the two disconnected tubes. The best results are achieved by generating a spline for each interconnecting slice, as this allows the algorithm to have maximum effect on each portion of the connecting tube, as seen in Figure 1;

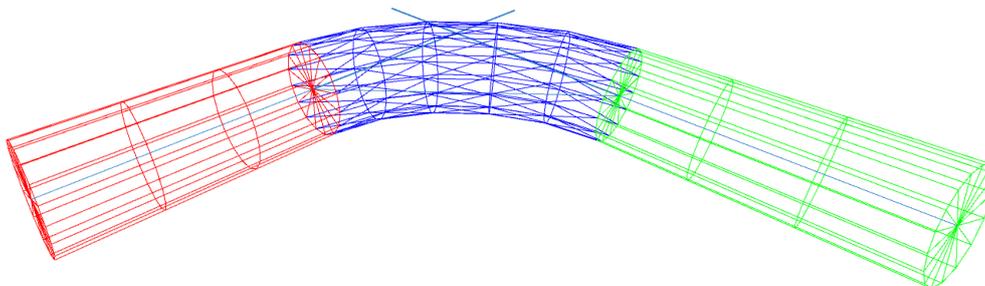


Figure 1: 4 point linear tangent

It is possible to use a bias in more interested forms. One example is using an exponential bias, rather than a linear bias. This can be used to produce an interesting corner shape when tubes are at 90° , which is not automatically possible with other methods of calculating the tangent, as seen in Figure 2.

3 Further Improvements

There are many ways this idea can be used and implemented, however as it stands, a more complex mu function based on pre-assigned data can offer more flexibility. For example the Kochanek-Bartels[4] spline function offers a much more complex set of parameters per point to define the shape of the curve, but the cost is that these must be defined manually.

The key benefit of this bias function is that it works based on the unit length of a local component of the curve. This allows you to have automatic control over the curve with respect to some value. Thus the tangent is not necessarily dependant on the distance between any particular points, such as with the Catmull-Rom method.

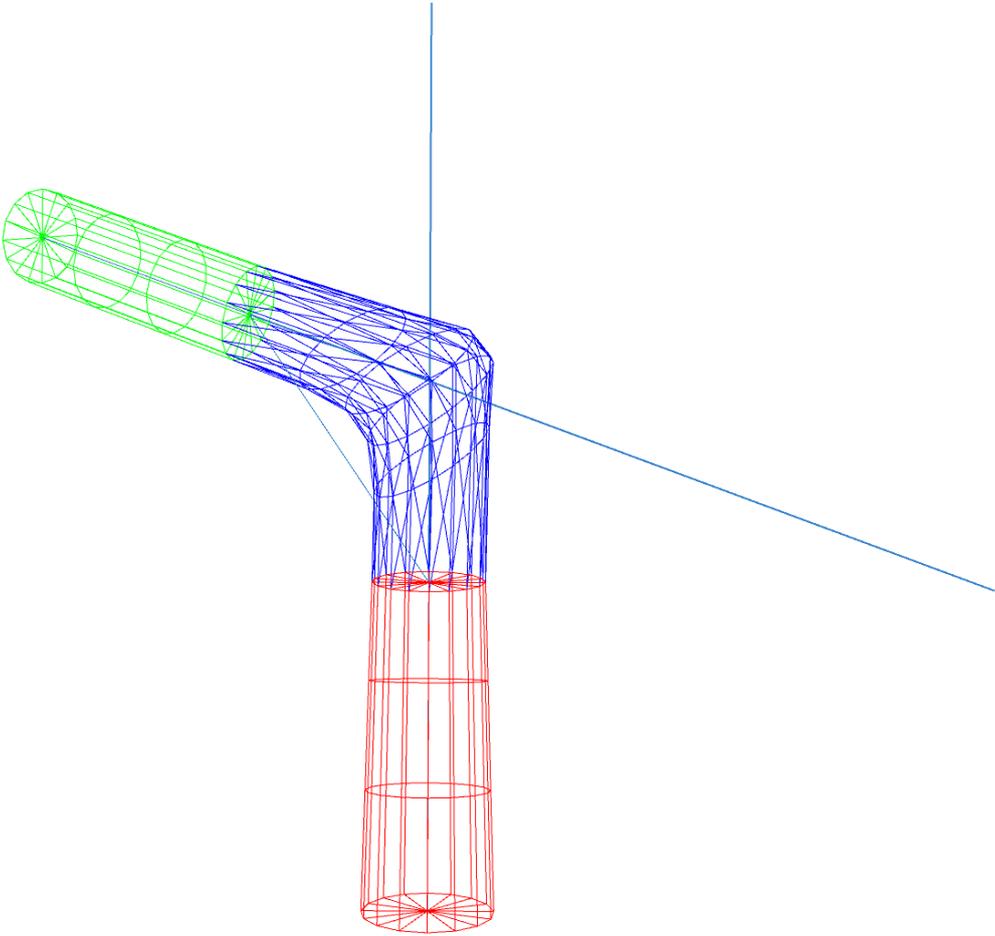


Figure 2: 4 point exponential tangent

There are no other available spline curves that allow for the integration of exponential distance as a parameter as a bias of the unit tangent, which makes this method unique in that regard. However, it may be simpler to use the Catmull-Rom method if these qualities are not needed. The Catmull-Rom has a lower computational cost, also.

References

- [1] Wikipedia. Cubic hermite spline — wikipedia, the free encyclopedia, 2007. [Online; accessed April-2007].
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- [3] Wikipedia. Catmull-rom spline — wikipedia, the free encyclopedia, 2007. [Online; accessed April-2007].
- [4] Wikipedia. Kochanek-bartels spline — wikipedia, the free encyclopedia, 2007. [Online; accessed April-2007].